LESSON 2-3 Conditional Statements

Target:_____

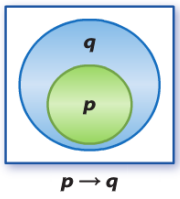
_____

1 If-Then Statements

A **conditional statement** is a statement that can be written in *if-then form*. The direction given below is an example of a conditional statement.

If you would like to speak to a representative, **then** you will press 0 now.

KeyConcept Conditional Statement

Words	Symbols	Model
An **if-then statement** is of the form *if p, then q.*	$p \rightarrow q$ read *if p then q*, or *p implies q*	
The **hypothesis** of a conditional statement is the phrase immediately following the word *if.*	p	
The **conclusion** of a conditional statement is the phrase immediately following the word *then.*	q	

2 Related Conditionals

There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form *if p, then q.*	$p \rightarrow q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \rightarrow p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \rightarrow \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \rightarrow \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

Identify the hypothesis and conclusion of each conditional statement.

1. If today is Friday, then tomorrow is Saturday.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Today is Friday.
Conclusion: Tomorrow is Saturday.

3. If two angles are supplementary, then the sum of the measures of the angles is 180.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word ***if***. The conclusion of a conditional statement is the phrase immediately following the word ***then***.

Hypothesis: Two angles are supplementary.
Conclusion: The sum of the measures of the angles is 180.

Write each statement in if-then form.

5. Sixteen-year-olds are eligible to drive.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis.
The word then is not part of the conclusion.

If you are sixteen years old, then you are eligible to drive.

7. The measure of an acute angle is between 0 and 90.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.
If the angle is acute, then its measure is between 0 and 90.

9. **WEATHER** Various kinds of precipitation form under different conditions. Write the three conditionals below in if-then form.

a. Moisture in the air condenses and falls to form rain.

b. Supercooled moisture in cumulonimbus clouds forms hail.

c. When the temperature is freezing in all or most of the atmosphere, precipitation falls as snow.

SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

a. If moisture in the air condenses and falls, then it rains.

b. If a cumulonimbus cloud has supercooled moisture, then hail forms.

c. If the temperature is freezing in all or most of the atmosphere, then precipitation falls as snow.

Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.

11. If you live in Charlotte, then you live in North Carolina.

SOLUTION:

The conditional is false. You could live in Charlotte, Michigan or Charlotte, North Carolina.

13. If an animal is spotted, then it is a Dalmatian.

SOLUTION:

False
The animal could be a leopard. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

15. If pigs can fly, then $2 + 5 = 7$.

SOLUTION:

The conditional statement "If pigs can fly, then $2 + 5 = 7$" is true. The hypothesis is false, since pigs cannot fly. A conditional with a false hypothesis is always true, so this conditional statement is true.

CCSS ARGUMENTS Write the converse, inverse, and contrapositive of each true conditional statement.
Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

17. All whole numbers are integers.

SOLUTION:

If a number is a whole number, then it is an integer.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If a number is an integer, then it is a whole number. False; Sample answer: –3.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If a number is not a whole number, then it i not an integer. False: Sample answer: –3.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If a number is not an integer, then it is not a whole number; true.

Identify the hypothesis and conclusion of each conditional statement.

19. If you lead, then I will follow.

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word *if*. The conclusion of a conditional statement is the phrase immediately following the word *then*.
Hypothesis: You lead
Conclusion: I will follow

Identify the hypothesis and conclusion of each conditional statement.

23. "If there is no struggle, there is no progress." (Frederick Douglass)

SOLUTION:

The hypothesis of a conditional statement is the phrase immediately following the word *if*. The conclusion of a conditional statement is the phrase immediately following the word *then*.
Hypothesis: There is no struggle
Conclusion: There is no progress

31. A right angle measures 90 degrees.

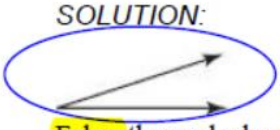
SOLUTION:

To write these statements in if-then form, identify the hypothesis and conclusion. The word if is not part of the hypothesis. The word then is not part of the conclusion.

If an angle is right, then the angle measures 90 degrees.

37. If an angle is acute, then it has a measure of 45.

SOLUTION:



False; the angle drawn is an acute angle whose measure is not 45. The hypothesis of the conditional is true, but the conclusion is false. This counterexample shows that the conditional statement is false.

Determine the truth value of each conditional statement. If true, explain your reasoning. If false, give a counterexample.

39. If an angle's measure is 25, then the measure of th angle's complement is 65.

SOLUTION:

The conditional statement "If an angle's measure is 25, then the measure of the angle's complement is 65." is true. When this hypothesis is true, the conclusion is also true, since an angle and its complement's sum is 90. So, the conditional statement is true.

Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is true or false. If a statement is false, find a counterexample.

47. If you live in Chicago, you live in Illinois.

SOLUTION:

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If you live in Illinois, then you live in Chicago. The converse is false.
Counterexample: You can live in Springfield.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If you do not live in Chicago, then you do not live in Illinois. The inverse is false.
Counterexample: You can live in Springfield.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If you do not live in Illinois, then you do not live in Chicago. The contrapositive is true.

51. All congruent segments have the same length.

SOLUTION:

If segments are congruent, then they have the same length.

The converse is formed by exchanging the hypothesis and conclusion of the conditional.
Converse: If segments have the same length, then they are congruent. The converse is true.

The inverse is formed by negating both the hypothesis and conclusion of the conditional.
Inverse: If segments are not congruent, then they do not have the same length. The inverse is true.

The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
Contrapositive: If segments do not have the same length, then they are not congruent. The contrapositive is true.