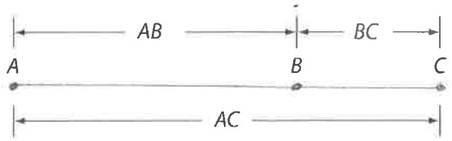
LESSON 2-7 Proving Segment Relationships

Target: I will be able to write proofs involving segment addition and segment congruence.

2.9 Segment Addition Postulate

Words If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

Symbols



2.2 Properties of Segment Congruence

Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Examples:

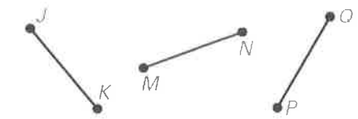
Justify each statement with a property, definition, postulate or theorem.

1. For any segment $\overline{EI}$, $\overline{EI} \cong \overline{EI}$
2. Given M is the midpoint of $\overline{TI}$, then $\overline{TM} \cong \overline{MI}$
3. If $\overline{JK} \cong \overline{YO}$ and $\overline{YO} \cong \overline{IM}$, then $\overline{JK} \cong \overline{IM}$
4. Given $\overline{AN} \cong \overline{HJ}$ then AN = HJ
5. If B is between endpoints A and E, then AB + BE=AE
6. If $\overline{PO} \cong \overline{KE}$ then $\overline{KE} \cong \overline{PO}$

1. Reflexive property of congruence
2. Midpoint Theorem
3. Transitive property of congruence
4. Definition of congruent segments
5. Segment Addition Postulate
6. Symmetric Property of congruence

7. **Example 1** *Transitive Property of Segment Congruence*

You can prove the Transitive Property of Segment Congruence as follows.

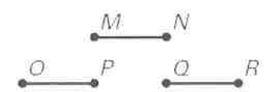


Given: $\overline{JK} \cong \overline{MN}$. $\overline{MN} \cong \overline{PQ}$
Prove: $\overline{JK} \cong \overline{PQ}$

Statements	Reasons
1. $\overline{JK} \cong \overline{MN}$. $\overline{MN} \cong \overline{PQ}$	1. Given
2. $JK = MN$. $MN = PQ$	2. Definition of congruent segments
3. $JK = PQ$	3. Transitive property of equality
4. $\overline{JK} \cong \overline{PQ}$	4. Definition of congruent segments

Complete the argument, giving a reason for each step.

8. Given: $OP = MN$. $MN = QR$
Prove: $\overline{OP} \cong \overline{QR}$



Statements	Reasons
1. $OP = MN$. $MN = QR$	1. Given
2. $OP = QR$	2. Transitive prop of equality
3. $\overline{OP} \cong \overline{QR}$	3. Definition of congruent segments

9. Prove that if $\overline{WY} \cong \overline{XZ}$ then $\overline{WX} \cong \overline{YZ}$

Given: $\overline{WY} \cong \overline{XZ}$
Prove: $\overline{WX} \cong \overline{YZ}$

Proof:

Statements	Reasons
1. $\overline{WY} \cong \overline{XZ}$	1. Given
2. $WY = XZ$	2. Definition of congruence segments
3. $WX + XY = WY$ $YZ + XY = XZ$	3. Segment addition postulate
4. $WX + XY = YZ + XY$	4. Substitution
6. $WX = YZ$	6. Subtraction property of equality
7. $\overline{WX} \cong \overline{YZ}$	7. Definition of congruent segments

10. **Given:** $AL = SK$

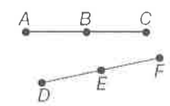
Prove: $AS = LK$

Statements	Reasons
1. $AL = SK$	1. ? Given
2. $LS = LS$	2. ? Reflexive prop. of equality
3. $AL + LS = SK + LS$	3. ? Addition prop. of equality
4. $AL + LS = AS$	4. ? Segment Addition postulate
5. $SK + LS = LK$	5. ? Segment Addition postulate
6. $AS = LK$	6. ? Substitution

11. **Given:** $\overline{AB} \cong \overline{DE}$
B is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{DF}$.

Prove: $\overline{BC} \cong \overline{EF}$

Proof:



Statements	Reasons
a. $\overline{AB} \cong \overline{DE}$	a. Given
B is the midpoint of $\overline{AC}$	
E is the midpoint of $\overline{DF}$	
b. $AB = DE$	b. Definition of congruent segments.
c. $AB = BC$	c. Definition of Midpoint
$DE = EF$	
d. $BC = DE$	d. Substitution
e. $BC = EF$	e. Substitution
f. $\overline{BC} \cong \overline{EF}$	f. Def. of $\cong$ seg's.

12. Given: T is the midpoint of $\overline{AM}$.

Prove: $AT = \frac{1}{2}AM$

STATEMENTS	REASONS
1. T is the m.p. of $\overline{AM}$.	1. Given
2. AT=TM	2. Definition of midpoint
3. AT + TM =AM	3. Segment Addition Postulate
4. AT + AT = AM	4. Substitution
5. 2AT = AM	5. Distributive Prop. / Simplify
6. AT = $\frac{1}{2}AM$	6. Multiplication property of equality

2-7 Practice (answer bank)	Substitution
$AB = BC$ $DE = EF$	$\overline{BC} \cong \overline{EF}$
Given	Substitution
Addition Property of Equality	$\overline{AB} \cong \overline{DE}$ B is the midpoint of $\overline{AC}$. E is the midpoint of $\overline{DF}$.
Definition of congruent segments	Definition of congruent segments
Substitution	Definition of congruent segments
Segment Addition postulate	Definition of congruent segments

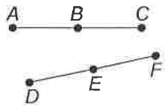
NAME _____ DATE _____ PERIOD _____

2-7 Practice

Proving Segment Relationships

Complete the following proof.

1. Given: $\overline{AB} \cong \overline{DE}$
B is the midpoint of $\overline{AC}$.
E is the midpoint of $\overline{DF}$.

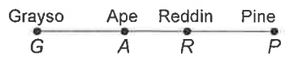


Prove: $\overline{BC} \cong \overline{EF}$

Proof:

Statements	Reasons
a. _____	**a.** Given
b. $AB = DE$	**b.** _____
c. _____	**c.** Definition of Midpoint
d. $BC = DE$	**d.** _____
e. $BC = EF$	**e.** _____
f. _____	**f.** _____

2. TRAVEL Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.



Given: $\overline{GA} \cong \overline{RP}$
Prove: $\overline{GR} \cong \overline{AP}$

Statements	Reasons
1. $\overline{GA} \cong \overline{RP}$	1.
2. $GA = RP$	2.
3. $GA + AR = AR + RP$	3.
4. $GR = GA + AR$, $AP = AR + RP$	4.
5. $GR = AP$	5.
6. $\overline{GR} \cong \overline{AP}$	6.