LESSON 5-1 Bisectors of Triangles

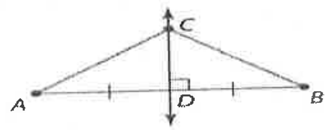
NOTES

Target: I will be able to identify and use perpendicular bisectors and angle bisectors in triangles.

Theorems Perpendicular Bisectors

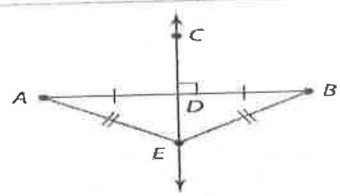
5.1 Perpendicular Bisector Theorem
If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Example: If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.



5.2 Converse of the Perpendicular Bisector Theorem
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Example: If $AE = BE$, then E lies on $\overline{CD}$, the $\perp$ bisector of $\overline{AB}$.



Example 1 Use the Perpendicular Bisector Theorems

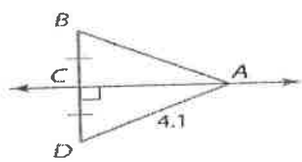
Find each measure.

a. AB

From the information in the diagram, we know that $\overleftrightarrow{CA}$ is the perpendicular bisector of $\overline{BD}$.

$AB = AD$ Perpendicular Bisector Theorem

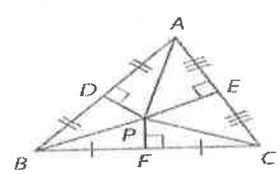
$AB = 4.1$ Substitution



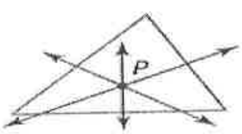
Theorem 5.3 Circumcenter Theorem

Words The perpendicular bisectors of a triangle intersect at a point called the *circumcenter* that is equidistant from the vertices of the triangle.

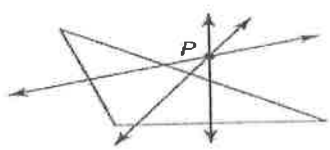
Example If P is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.



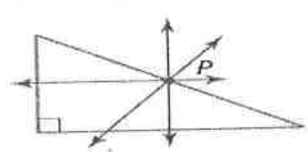
The circumcenter can be on the interior, exterior, or side of a triangle.



acute triangle



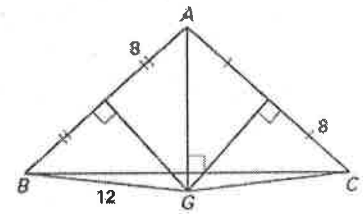
obtuse triangle



right triangle

EXAMPLE 2 Use the Circumcenter Theorem

The perpendicular bisectors of $\triangle ABC$ meet at point G. Find GA.



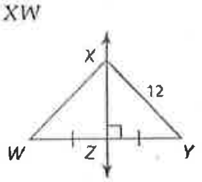
From the Circumcenter Theorem, G is called the circumcenter which is equidistant from the vertices. Thus, $\overline{BG} \cong \overline{AG}$

Therefore, $GA = 12$

Practice:

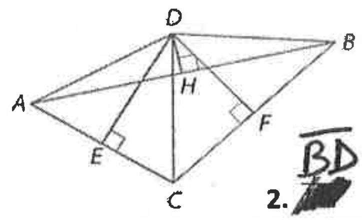
Find each measure.

1. XW



$XW = XY = 12$

Point D is the circumcenter of $\triangle ABC$. List any segment(s) congruent to each segment.



2. [redacted] $\overline{BD}$

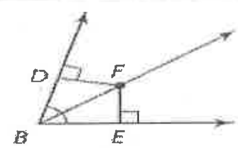
$\overline{BD} \cong \overline{AD} \cong \overline{CD}$

Theorems Angle Bisectors

5.4 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

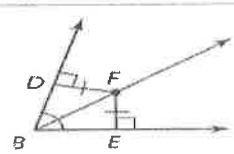
Example: If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$, and $\overline{FE} \perp \overrightarrow{BE}$, then $DF = FE$.



5.5 Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

Example: If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.



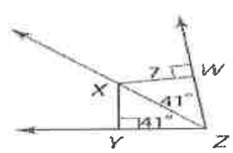
Example 3 Use the Angle Bisector Theorems

Find each measure.

a. XY

$XY = XW$ Angle Bisector Theorem

$XY = 7$ Substitution



c. SP

$SP = SM$ Angle Bisector Theorem

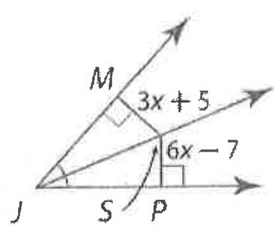
$6x - 7 = 3x + 5$ Substitution

$3x - 7 = 5$ Subtract $3x$ from each side.

$3x = 12$ Add 7 to each side.

$x = 4$ Divide each side by 3.

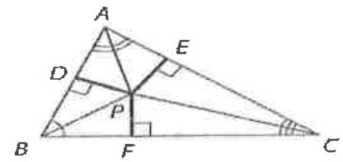
$SP = 6(4) - 7 = 17$



Theorem 5.6 Incenter Theorem

Words The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle.

Example If P is the incenter of $\triangle ABC$, then $PD = PE = PF$.

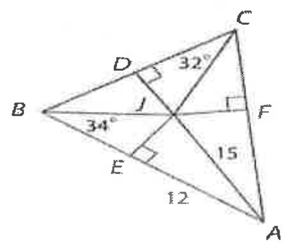


Example 4 Use the Incenter Theorem

Find each measure if J is the incenter of $\triangle ABC$.

a. JF

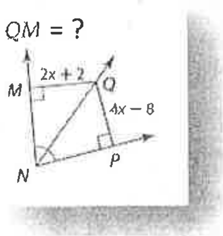
By the Incenter Theorem, since J is equidistant from the sides of $\triangle ABC$, $JF = JE$. Find JF by using the Pythagorean Theorem.



$JE^2 + EA^2 = JA^2$	Pythagorean Theorem
$JE^2 + 12^2 = 15^2$	Substitution
$JE^2 + 144 = 225$	$12^2 = 144$ and $15^2 = 225$.
$JE^2 = 81$	Subtract 144 from each side.
$JE = 9$ so $JF = 9$	Take the square root of each side.

Practice:

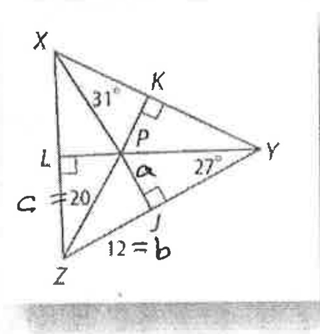
3. $QM = ?$



$QM = QP$
$2x+2 = 4x-8$
$2 = 2x-8$
$10 = 2x$
$5 = x$
$QM = 2(5)+2 = 12$

If P is the incenter of $\triangle XYZ$, find each measure.

4. PK



$PK = PL = PJ$
$a^2+b^2=c^2$
$a^2+12^2=20^2$
$a^2+144=400$
$a^2=256$
$a=16$
$PK=16$

Mr. B is designing a kitchen, Mr. B has found out the locations of the Sink K, the Stove S, and the Refrigerator R. Mr. B wants to put a small island in the kitchen equidistant from each vertex, where should Mr. B place the island? at the circumcenter, where the perp. bisectors intersect.