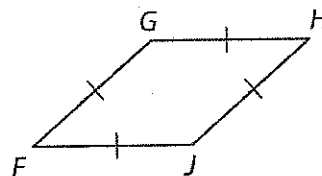


6-5 Rhombi and Squares

Target: I will be able to use properties of rhombi and squares to solve problems.

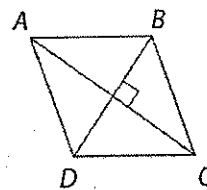
1 Properties of Rhombi and Squares A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



Theorems Diagonals of a Rhombus

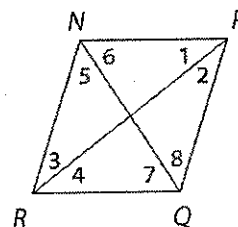
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If $\square ABCD$ is a rhombus, then $\overline{AC} \perp \overline{BD}$.

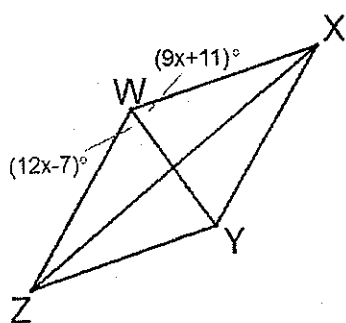


6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

Example If $\square NPQR$ is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.



1. WXYZ is a rhombus. What is the measure of $\angle WXY$?

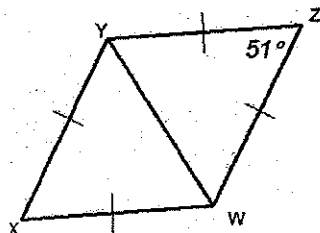


$$\begin{array}{r} 12x - 7 = 9x + 11 \quad (\text{Thm. 6.16}) \\ -9x \quad \quad -9x \\ \hline 3x - 7 = 11 \\ +7 \quad +7 \\ \hline 3x = 18 \\ \frac{3x}{3} = \frac{18}{3} \\ x = 6 \end{array}$$

$$m\angle ZWX = 12(6) - 7 + 9(6) + 11 = 130$$

$$\begin{aligned} m\angle WXY &= 180 - m\angle ZWX \\ m\angle WXY &= 180 - 130 = 50^\circ \end{aligned}$$

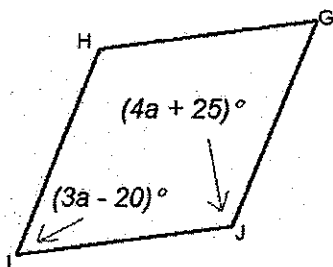
2. WXYZ is a rhombus. What is the measure of $\angle XYW$?



$$m\angle XYZ = 180 - 51 = 129^\circ$$

$$m\angle XYW = \frac{1}{2}(129) = 64.5^\circ$$

3. GHIJ is a rhombus. Find the value of a.



$$m\angle I + m\angle J = 180$$

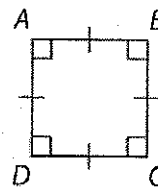
$$3a - 20 + 4a + 25 = 180$$

$$7a + 5 = 180$$

$$7a = 175$$

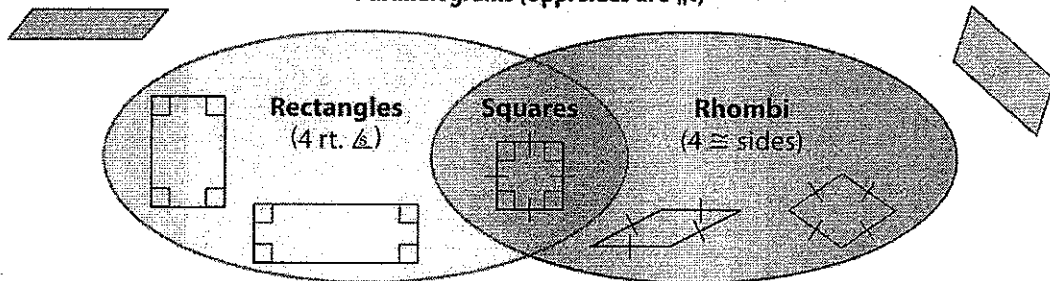
$$a = 25$$

A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.



ConceptSummary Parallelograms

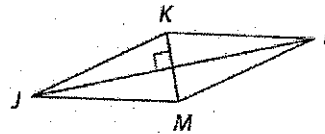
Parallelograms (Opp. sides are \parallel .)



Theorems Conditions for Rhombi and Squares

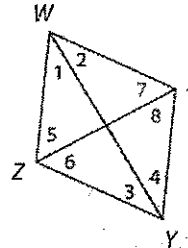
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



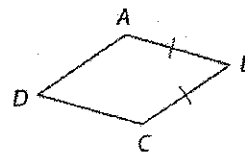
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

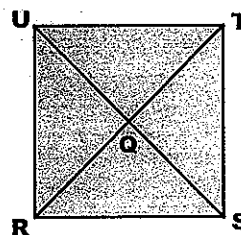
Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

4. For the square below, $UQ = 7x + 14$ and $RT = 18x - 4$, find US .

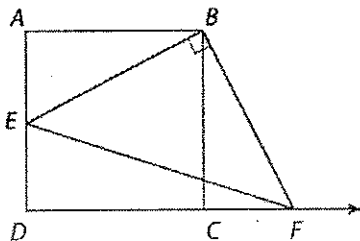
$$\begin{aligned} RT &= 2 \cdot UQ \\ 18x - 4 &= 2(7x + 14) \\ 18x - 4 &= 14x + 28 \\ 4x - 4 &= 28 \\ 4x &= 24; x &= 6 \end{aligned}$$



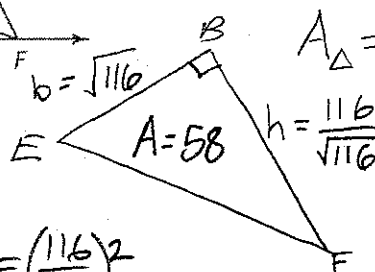
$$\begin{aligned} US &= RT = 18(6) - 4 \\ US &= 104 \end{aligned}$$

5. The area of square ABCD is 100 square units and the area of $\triangle EBF$ is 58 square units. If

$\overline{EB} \perp \overline{BF}$ and $\overline{AE} = 4$, find the length of \overline{CF} . $A = S^2 = 100$; $S = \sqrt{100} = 10$



$$\begin{aligned} 10^2 + 4^2 &= C^2 \\ 100 + 16 &= C^2 \\ \sqrt{116} &= C \end{aligned}$$



$$\begin{aligned} A_{\triangle} &= \frac{1}{2} b \cdot h \rightarrow 58 = \frac{1}{2} \sqrt{116} \cdot h \\ 116 &= \sqrt{116} \cdot h \\ \frac{116}{\sqrt{116}} &= h \end{aligned}$$



$$a^2 + 10^2 = \left(\frac{116}{\sqrt{116}}\right)^2$$

$$a^2 + 100 = 116 \rightarrow a = \sqrt{116 - 100} = \sqrt{16} = 4$$

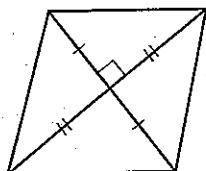
6-5 practice A / 6-5 practice 1

| | | | | |
|----------------------------------|--|--|--|----------------------------------|
| 44 | B, D | rhombus; diagonals are ⊥ but not ≅. | rectangle | rectangle; diagonals are ≅ |
| rectangle; ∠s are right ∠s | 242 | parallelogram | A, B, C, D | C, D |
| 13 | square; diagonals are ⊥ and ≅ | DF | 5 | 66.5 |
| rhombus | A, B, C, D | 11 | square; adj. sides are ≅ & ∠s are right ∠s | D |
| 6 3 | rhombus; adjacent sides are ≅ | B, D | square | 16 |

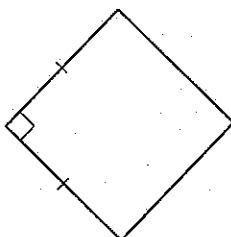
Practice A

Each figure is a parallelogram. Identify the special type and explain your reasoning.

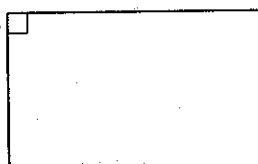
1.



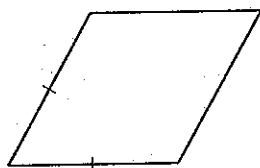
2.



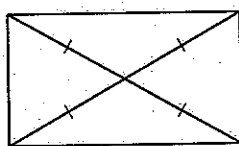
3.



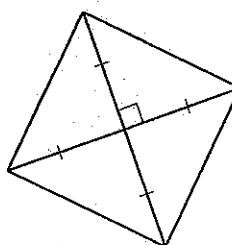
4.



5.



6.



Match the properties of a quadrilateral with all of the types of quadrilateral which have that property.

- | | |
|--|------------------|
| 7. The diagonals are congruent. | A. Parallelogram |
| 8. Both pairs of opposite sides are congruent. | B. Rectangle |
| 9. Both pairs of opposite sides are parallel. | C. Rhombus |
| 10. All angles are congruent. | D. Square |
| 11. All sides are congruent. | |
| 12. Diagonals bisect the angles. | |

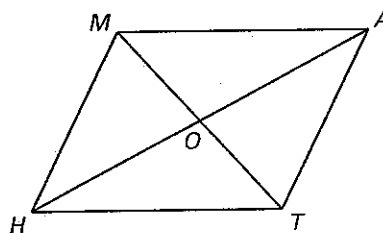
MATH is a parallelogram with diagonals intersecting at **O**. Identify the type depending upon the given conditions.

13. $\overline{MT} \perp \overline{AH}$

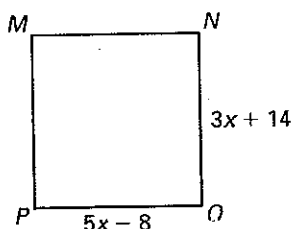
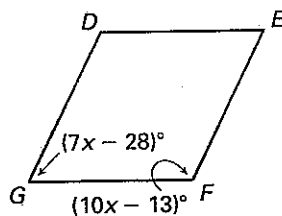
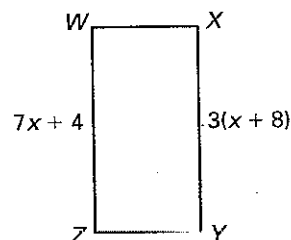
14. $\overline{MT} \cong \overline{AH}$

15. $\overline{MA} \perp \overline{AT}, \overline{AM} \cong \overline{MH}$

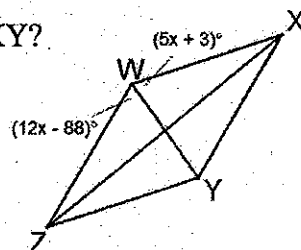
16. $\overline{MO} \cong \overline{OT}, \overline{AO} \cong \overline{OH}$



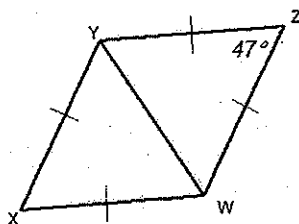
Find the value of x .

17. $MNOP$ is a square.18. $DEFG$ is a rhombus.19. $WXYZ$ is a rectangle.

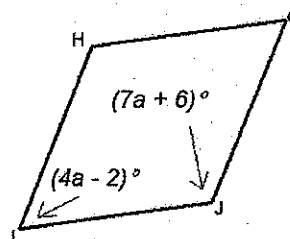
1. WXYZ is a rhombus. What is the measure of $\angle WXY$?



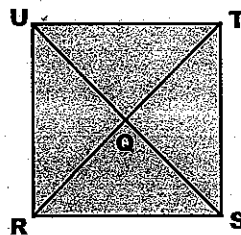
2. WXYZ is a rhombus. What is the measure of $\angle XYW$?



3. GHIJ is a rhombus. Find the value of a .



4. For the square below, $UQ = 6x + 19$ and $RT = 15x - 13$, find US .



5. The area of square ABCD is 81 square units and the area of $\triangle EBF$ is 45 square units. If $\overline{EB} \perp \overline{BF}$ and $\overline{AE} = 3$, find the length of \overline{CF} .

